SADLER UNIT 4 MATHEMATICS METHODS

WORKED SOLUTIONS

Chapter 5 Random sampling

Exercise 5A

Question 1

a Likely to introduce bias.

b Likely to introduce bias.

c Not likely to introduce bias.

d Not likely to introduce bias.

e Not likely to introduce bias.

Question 2

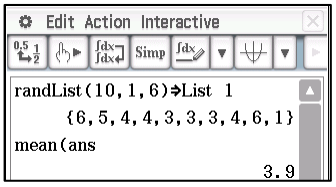
497 in total

Based on percentage:

Age	Number		No. in Committee
$x < 20$	100	$\frac{100}{497} \times 10$	2
$20 \leq x < 30$	154	$\frac{154}{497} \times 10$	3
$30 \leq x < 40$	175	$\frac{175}{497} \times 10$	4
$x \geq 40$	68	$\frac{68}{497} \times 10$	1

Question 3

a Answers will differ but ClassPad instructions are shown

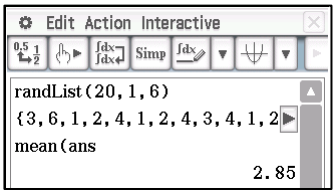


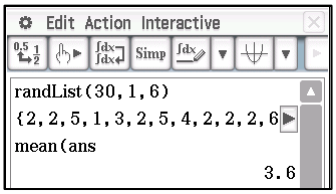
20 rolls $\bar{x} = 2.85$

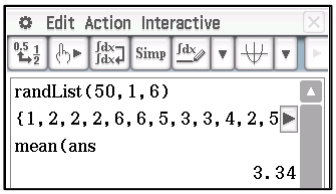
30 rolls $\bar{x} = 3.6$

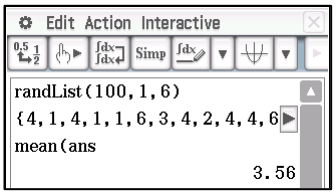
50 rolls $\bar{x} = 3.34$

100 rolls $\bar{x} = 3.56$









Question 4

80 students from 1744 in total

Year

8	$\frac{420}{1744} \times 80$	≈ 19.27	19
9	$\frac{407}{1744} \times 80$	≈ 18.67	19
10	$\frac{389}{1744} \times 80$	≈ 17.8	18
11	$\frac{270}{1744} \times 80$	≈ 12.4	12
12	$\frac{258}{1744} \times 80$	≈ 11.8	12

Question 5

$$\frac{34}{x} = \frac{5}{28}$$

$x \approx 190$ frogs

Question 6

$$\frac{64}{x} = \frac{7}{83}$$

$x \approx 760$

Question 7

$$\frac{123}{x} = \frac{6}{154}$$
$$x = \frac{123 \times 154}{6}$$
$$= 3157$$
$$\therefore \approx 3200$$

Question 8

Shane : 12 trials

Christine : 150 trials

$$\mathrm{E}(x) = \frac{1 + 2 + ... + 6}{6}$$
$$\approx 3.5$$

$\therefore$ 3.48 belongs to Christine as she is closest to $\mathrm{E}(x)$ due to more trials.

Question 9

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\mathrm{E}(x) = 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + ... + 12 \times \frac{1}{36}$$
$$= 7$$

Portia : 12 trials

Horace : 150 trials

$\mu_1 = 7.17$, $\mu_2 = 6.42$

μ_1 is closer to 7 and we expect a simulation with more trials to be closer to $\mathrm{E}(x)$.

$\therefore$ Horace : 7.17

Question 10

$$\frac{7}{1345} = \frac{x}{1\,235\,067}$$

$$x = 6428$$

$\therefore$ 6500 people

Question 11

Refer to textbook answer.

Question 1

$$\log 100 - \ln(e^{-5})$$
$$= 2 - (-3)\ln e$$
$$= 5$$

Question 2

$$P = 9e^{(t+1)}$$
$$\frac{P}{9} = e^{t+1}$$
$$\ln\left(\frac{P}{9}\right) = (t+1)\ln e$$
$$\ln\left(\frac{P}{9}\right) = t+1$$
$$t = \ln\left(\frac{P}{9}\right) - 1$$

a
$$t = \ln\left(\frac{100}{9}\right) - 1$$
$$= 1.996$$

b
$$t = \ln\left(\frac{3600}{9}\right) - 1$$
$$= 4.991$$

c
$$t = \ln\left(\frac{9e^3}{9}\right) - 1$$
$$= 3\ln e - 1$$
$$= 2$$

Question 3

a $\log ab$

$= \log a + \log b$

$= p + q$

b $\log\left(\frac{a}{b}\right)$

$= \log a - \log b$

$= p - q$

c $\log(a^2 b^3)$

$= \log a^2 + \log b^3$

$= 2\log a + 3\log b$

$= 2p + 3q$

d $\log \sqrt{a}$

$= \log a^{\frac{1}{2}}$

$= \frac{1}{2}\log a$

$= \frac{1}{2}p$

e $\ln a$

$= \frac{\log a}{\log e}$

$= \frac{p}{\log e}$

f $\log_5 b^2$

$= \frac{2\log b}{\log 5}$

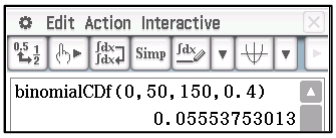
$= \frac{2q}{\log\left(\frac{10}{2}\right)}$

$= \frac{2q}{\log 10 - \log 2}$

$= \frac{2q}{1 - \log 2}$

Question 4

$$\mathrm{E}(X) = np = 60$$
$$\mathrm{Var}(X) = np(1-p) = 36$$
$$60(1-p) = 36$$
$$1-p = 0.6$$
$$p = 0.4$$
$$n \times 0.4 = 60$$
$$n = \frac{60}{0.4}$$
$$= 150$$



$$\mathrm{P}(X \leq 50) = 0.056$$

Question 5

$$\frac{dy}{dx} = x \times \frac{1}{x} + \ln 5x \times 1$$
$$= 1 + \ln(5x)$$

Question 6

$$\frac{dy}{dx} = 2(\log_e x) \times \frac{1}{x}$$
$$= \frac{2\log_e x}{x}$$

Question 7

$$\frac{dy}{dx} = x^2 \times \frac{1}{x} + \ln x \times 2x$$
$$= x + 2x\ln x$$
$$= x(1 + 2\ln x)$$

Question 8

$$\frac{dy}{dx} = 2(3+\ln x)\times\frac{1}{x}$$
$$= \frac{2(3+\ln x)}{x}$$

Question 9

$$\frac{dy}{dx} = \frac{-2}{x^2} + 2\times\frac{1}{x}$$
$$= \frac{2}{x} - \frac{2}{x^2}$$
$$= \frac{2x-2}{x^2}$$
$$= \frac{2(x-1)}{x^2}$$

Question 10

$$y = (\ln x)^{-1}$$
$$\frac{dy}{dx} = -1(\ln x)^{-2}\times\frac{1}{x}$$
$$= \frac{-1}{x(\ln x)^2}$$

Question 11

$$f(x) = x^3 \ln x$$
$$f'(x) = x^3\times\frac{1}{x} + \ln x\times 3x^2$$
$$= x^2 + 3x^2 \ln x$$
$$f''(x) = 2x + 3x^2\times\frac{1}{x} + \ln x\times 6x$$
$$= 2x + 3x + \ln x 6x$$
$$= 5x + 6x\ln x$$

Question 12

a $x = 9\ln(t+1) - 4t$

$$V = \frac{dx}{dt} = \frac{9}{t+1} - 4 = 0$$

$$\frac{9}{t+1} = 4$$

$$t+1 = \frac{9}{4}$$

$$t = 1.25$$

b $$a = \frac{dv}{dt} = \frac{-9}{(t+1)^2}$$

$$\frac{-9}{(t+1)^2} = \frac{9}{(t+1)} - 4$$

By classpad

$$t = -1.75,\ 2$$

$$\therefore t = 2\ (t \geq 0)$$

Question 13

a $$\int_1^3 k(4-x)\ dx$$

$$= k\left[4x - \frac{x^2}{2}\right]_1^3$$

$$= k\left((12 - 4.5) - \left(4 - \frac{1}{2}\right)\right)$$

$$= 4k$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b $$\mathrm{E}(X) = \int_1^3 xf(x)\ dx$$

$$= \int_1^3 \frac{x}{4}(4-x)\ dx$$

$$= \frac{11}{6}$$

c $\text{Var}(X) = \int_1^3 \frac{1}{4}(4-x)\left(x-\frac{11}{6}\right)^2 dx$

$= \frac{1}{4}\int_1^3 (4-x)\left(x-\frac{11}{6}\right)^2 dx$

$= \frac{11}{36}$

d $\text{SD}(X) = \sqrt{\frac{11}{36}}$

$= \frac{\sqrt{11}}{6}$

e $\int_1^x \left(\frac{4-t}{4}\right) dt$

$= \int_1^x \left(1-\frac{t}{4}\right) dt$

$= \left[t-\frac{t^2}{8}\right]_1^x$

$= x-\frac{x^2}{8}-\left(1-\frac{1}{8}\right)$

$= x-\frac{x^2}{8}-\frac{7}{8}$

$= \frac{1}{8}(8x-x^2-7)$

$$\text{P}(X \le x) = \begin{cases} 0 & \text{for } x < 1 \\ -\frac{1}{8}(x^2-8x+7) & \text{for } 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

Question 14

$$\frac{127}{x} = \frac{3}{89}$$

$$x = 3767\tfrac{2}{3}$$

First estimate 3768

$$\frac{127}{x} = \frac{4}{99}$$

$$x = 3143.25$$

Second estimate 3143

$$\frac{3768 + 3143}{2} = 3455.5$$

Approximately 3500 birds

See textbook answer for suggestions to problems associated with approach.

Question 15

a F

b F

c F

d F

e F

f F

g F

h F

i T

j T

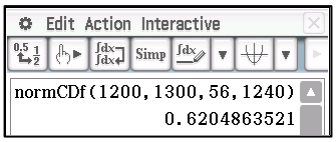
k T

l T

$\therefore$ Only i, j, k and l are always true.

Question 16

$P(1200 < X < 1300) = 0.0625$



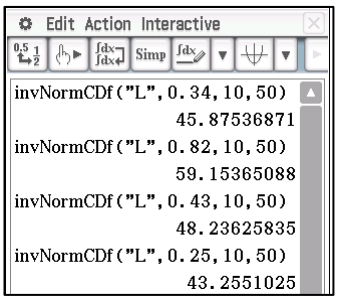
Question 17

a 45.9

b 59.2

c 48.2

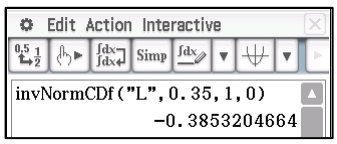
d 43.3



Question 18

a $P(X < 28) = 0.35$

28 is 0.385 standard deviations below the mean.



b

$$z = \frac{x - \mu}{\sigma}$$

$$-0.385 = \frac{28 - \mu}{5.74}$$

$$\mu = 28 + 0.385 \times 5.74$$

$$= 30.21$$